MEXANIKA MASALALARINI LAGRANJ TENGLAMASINI QO‘LLAGAN HOLDA YECHISH

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Annotatsiya: *Ushbu maqolada eng kichik ta’sir prinsipi va undan kelib chiqadigan Lagrang tenglamasining ahamiyati, mexanika masalalarini Lagranj tenglamasidan foydalangan holda yechish yo‘llarini o‘rganish maqsadida namuna masalalar yechib ko’rsatilgan.*

Kalit so’zlar: *mexanika; harakat tenglamasi; Lagranj funksiyasi; Lagranj tenglamasi; umumlashgan koordinata; eng kichik ta’sir prinsipi; ta’sir integrali; xususiy hosila.*

SOLVING MECHANICAL PROBLEMS USING THE LAGRANGE EQUATION

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Annotation: *This article examines the principle of least action and the meaning of the Lagrange equation resulting from it, in order to study ways to solve problems in mechanics using the Lagrangian equation.*

Key words: *mechanics; equation of motion; Lagrange function; Lagrange equation; generalized coordinate; principle of least action; Integral of action; partial derivative.*

Аннотация: *В этой статье рассматривается принцип наименьшего действия и значение уравнения Лагранжа, вытекающего из него, с целью изучения способов решения задач механики с использованием уравнения Лагранжа.*

Ключевые слова: *механика; уравнение движения; функция Лагранжа; уравнение Лагранжа; обобщенная координата; принцип наименьшего действия; Интеграл действия; частная производная.*

Ko‘pchiligimizga ma’lumki, oliy ta’lim muassasalarining “Fizika” yoki “Fizika va astronomiya” mutaxassisliklarida “Nazariy fizika”ning I qismi “Nazariy mexanika” yoki “Klassik mexanika” deb ataluvchi bo‘lim o‘qitiladi. Bu bo‘limda biror mexanik hodisa bilan bog‘liq forlmulalar yoki mexanik masalalar Nyuton mexanikasida o‘rganilgan Nyuton qonunlar bilan emas, balki nazariy yo‘l bilan hamda Lagranj va Gamilton formulalalari bilan tushuntiriladi va ishlanadi. Lekin, aksariyat talabalarda faqat Nyuton mexanikasi hamda Nyuton qonunlari bilan mexanika masalalarini tushuntirish va yechish ko‘nikmalari shakllangan. Aynan shuning uchun ham ular har qanday mexanika bo‘limiga tegishli muammo yoki masalani o‘zlari o‘rgangan usulda bartaraf etishga harakat qilishadi, nazariy yo‘l bilan esa juda kamchilik talabalar shug‘ullanishadi. Shu maqsadda mexanika masalalarini nazariy yo‘l bilan tushuntirish va masalalarni Lagranj yoki Gamilton tenglamalari bilan ishlashni o‘rganish va bu bo‘yicha ko‘nikma va malakalar shakllantirish ayni muddaodir. Mazkur maqola ana shunday muammoni hal etishga bag‘ishlanadi.

Ma’lumki, nazariy mexanikada ixtiyoriy sharoitda harakatlanayotgan nuqtaviy jism vaqtning t_1 paytida holati A nuqtada, t_2 paytida esa B nuqtada bo‘lsa, A va B nuqtani tutashtiruvchi cheksiz ko‘p trayektoriyalar ichida aynan bitta trayektoriyani tanlashi sababini ta’sir integralining minimallik sharti asosida tekshiriladi.

$$s = \int_{t_1}^{t_2} L(q,\dot{q},t)\,dt \to \min \tag{I}$$

Bunda, $L(q,\dot{q},t)$ – umumlashgan koordinata, umumlashgan tezlik hamda vaqtning funksiyasi bo‘lgan Lagranj funksiyasi bo‘lib, u energiya o‘lchamiga ega.

$$L = K - U = \frac{1}{2}m\dot{q}^2 - U \tag{II}$$

Bunda, U – jism yoki nuqtaning potensial energiyasi; K – jism yoki nuqtaning kinetik energiyasi bo‘lib, (II) tenglamada faqat ilgarilanma harakat uchun $K = \frac{1}{2}m\dot{q}^2$ ko‘rinishda berilgan. Aslida, aylanma harakat yoki tebranma harakat qilgan hollar uchun aylanma va tebranma harakatlarga mos kinetik energiyalarini qo‘yish joiz bo‘ladi.

(I) tenglamadagi ta’sir integralining minimallik shartidan Lagraj tenglamasi kelib chiqariladi.

$$\frac{\partial L}{\partial q}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)=0 \qquad \textbf{(III)}$$

Mana shu (III) tenglama xuddi Nyuton mexanikasidagi *F=ma* formula kabi nazariy mexanikaning matematik apparati hisoblanadi.

Endi esa mexanikaning kinematika va dinamika boblarida ko‘p uchraydigan masalalardan tanlab, ularni Lagranj tenglamasi yordamida yechishga harakat qilib ko‘raylik.

Masala 1 (prujinali mayatnik masalasi). Bikrligi *k* bo‘lgan prujinaga *m* massali jism osilgan. Bu jismni muvozonat hoaltdan chiqarish orqali tebranma harakatga keltirilsa, u holda harakat tenglamasi qanday ko‘rinishda bo‘ladi? Harakat tenglamasini Lagranj tenglamasi yordamida keltirib chiqaring.

Yechish: Deformatsiyalanmagan prujinaning uchini koordinata boshi deb tanlab, deformatsiya kattaligi *x* bo‘lganda prujinaga mahkamlangan sharcha kinetik energiyasini $K=\frac{1}{2}m\dot{x}^2$ deb, potensial energiyasini esa $U=\frac{1}{2}kx^2$ deb olamiz. Natijada, Lagranj funksiyasi

$$L=K-U=\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2 \qquad \textbf{(1.1)}$$

ifodasi hosil bo‘ladi. (III) Lagranj tenglamasidagi xususiy hosilalarni topamiz.

$$\frac{\partial L}{\partial q}=\frac{\partial L}{\partial x}=\frac{\partial}{\partial x}\left(\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2\right)=-kx \qquad \textbf{(1.2)}$$

$$\frac{\partial L}{\partial \dot{q}}=\frac{\partial L}{\partial \dot{x}}=\frac{\partial}{\partial \dot{x}}\left(\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2\right)=m\dot{x} \qquad \textbf{(1.3)}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)=\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)=\frac{d}{dt}(m\dot{x})=m\ddot{x} \qquad \textbf{(1.4)}$$

Endi esa (1.2) va (1.4) ifodalarni (III) tenglamaga qo‘yamiz.

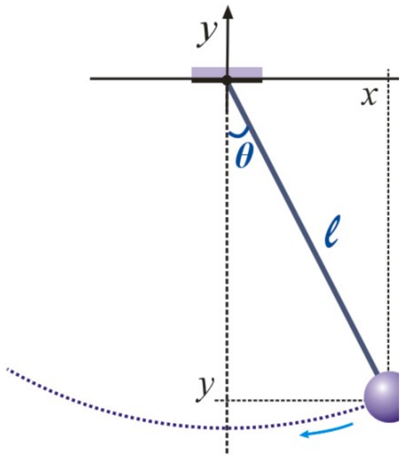
$$-kx=m\ddot{x},\ \rightarrow\ \ddot{x}+\frac{k}{m}x=0,\quad \ddot{x}+\omega^2x=0 \qquad \textbf{(1.5)}$$

Hosil bo‘lgan tenglama, tebranayotgan prujinaga mahkamlangan sharchaning harakat tenglamasi (aniqrog‘i harakatining differensial tenglamasi) deyiladi. Bu differensial tenglamaning (umumiy qilib aytganda barcha erkin garmonik tebranishlarning) yechimnini umumiy holda

$$x = A\sin(\omega t + \varphi_0) \quad \text{yoki} \quad x = A\cos(\omega t + \varphi_0) \qquad \textbf{(1.6)}$$

ko‘rinishda bo‘lishini Nyuton mexanikasidan ham yaxshi bilamiz. Shu boisdan ham bundan keyingi masalalarda (III) Lagranj tenglamasidan foydlaanib faqat harakat (differensial) tenglamasining o‘zini keltirib chiqarish bilan cheklanamiz.

Masala 2 (matematik mayatnik masalasi). Uzunligi ℓ bo‘lgan ipga ozilgan m massali sharchaning kichik tebranishlar bilan tebranishiga matematik mayatnik deyiladi. Bu matematik mayatnik uchun harakat tenglamasini Lagranj tenglamasi yordamida keltirib chiqaring.



Yechish: Mayatnik osilgan nuqtani koordinata boshi deb olib, bu nuqtadan Ox va Oy o‘qlarini o‘tkazamiz (rasmga qarang). Mayatnik ixtiyoriy vaziyatda turganida vertikal bila θ burchak tashkil qiladi. Bunda korrdinatalar va tezliklar

$$\begin{cases} x = \ell\sin\theta \\ y = -\ell\cos\theta \end{cases} \quad \text{va} \quad \begin{cases} \dot{x} = \ell\cos\theta\cdot\dot{\theta} \\ \dot{y} = \ell\sin\theta\cdot\dot{\theta} \end{cases} \qquad \textbf{(2.1)}$$

bo‘ladi. Bunda kinetik va potensial energiyalar

$$K = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) = \frac{1}{2}m\ell^2\left(\cos^2\theta + \sin^2\theta\right)\dot{\theta}^2 = \frac{1}{2}m\ell^2\dot{\theta}^2 \qquad \textbf{(2.2)}$$

$$U = mgy = -mg\ell\cos\theta \qquad \textbf{(2.3)}$$

bo‘ladi. Tebranayotgan matematik mayatnik uchun Lagranj funksiyasi

$$L = K - U = \frac{1}{2}m\ell^2\dot{\theta}^2 + mg\ell\cos\theta \qquad \textbf{(2.4)}$$

bo‘ladi. (III) Lagranj tenglamasidagi xususiy hosilalarni topamiz.

$$\frac{\partial L}{\partial q} = \frac{\partial L}{\partial \theta} = \frac{\partial}{\partial \theta}\left(\frac{1}{2}m\ell^2\dot{\theta}^2 + mg\ell\cos\theta\right) = -mg\ell\sin\theta \qquad \textbf{(2.5)}$$

$$\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial \dot{\theta}} = \frac{\partial}{\partial \dot{\theta}}\left(\frac{1}{2}m\ell^2\dot{\theta}^2 + mg\ell\cos\theta\right) = m\ell^2\dot{\theta} \qquad \textbf{(2.6)}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{d}{dt}\left(m\ell^2\dot{\theta}\right) = m\ell^2\ddot{\theta} \qquad \textbf{(2.7)}$$

Endi esa (2.5) va (2.7) ifodalarni (III) tenglamaga qo‘yamiz.

$$-mg\ell\sin\theta - m\ell^2\ddot{\theta} = 0, \ \rightarrow \ \ddot{\theta} + \frac{g}{\ell}\sin\theta = 0 \quad \textbf{(2.8)}$$

Kichik tebranishlarda *sinθ≈θ* ekanligini e'tiborga olsak,

$$-mg\ell\sin\theta - m\ell^2\ddot{\theta} = 0, \ \rightarrow \ \ddot{\theta} + \frac{g}{\ell}\sin\theta = 0 \quad \textbf{(2.9)}$$

bo'ladi. Hosil bo'lgan tenglama, kichik tebranishlar bilan tebranayotgan matematik mayatnik harakatining differensial tenglamasi hisoblanadi. Bu tenglamani yechib chiqish orqali *θ=θ(t)* harakat tenglamasini olish mumkin.

Masala 3 (yuqoriga tik otilgan jism masalasi). Yer sirtidan biror boshlang'ich tezlik bilan vertikal holda tik yuqoriga otilgan jism uchun harakat tenglamasini Lagranj tenglamasi yordamida keltirib chiqaring.

Yechish: Yer sirtidan boshlang'ich tezlik bilan tik holda yuqoriga qarab otilgan jism harakati faqat *Oy* o'qi bo'yicha sodir bo'lgani uchun jismning tezlik va koordinata o'zgarishini faqat shu o'qda olamiz. Qulaylik uchun koordinata boshini yer sirtida olamiz. Jism ixtiyoriy *y* koordinatada bo'lganida uning kinetik va potensial energiyalar

$$K = \frac{1}{2}m\dot{y}^2 \quad \text{va} \quad U = mgy \quad \textbf{(3.1)}$$

bo'ladi. Bu jism uchun Lagranj funksiyasi

$$L = K - U = \frac{1}{2}m\dot{y}^2 - mgy \quad \textbf{(3.2)}$$

bo'ladi. (III) Lagranj tenglamasidagi xususiy hosilalarni topamiz.

$$\frac{\partial L}{\partial q} = \frac{\partial L}{\partial y} = \frac{\partial}{\partial y}\left(\frac{1}{2}m\dot{y}^2 - mgy\right) = -mg \quad \textbf{(3.3)}$$

$$\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial \dot{y}} = \frac{\partial}{\partial \dot{y}}\left(\frac{1}{2}m\dot{y}^2 - mgy\right) = m\dot{y} \quad \textbf{(3.4)}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) = \frac{d}{dt}(m\dot{y}) = m\ddot{y} \quad \textbf{(3.5)}$$

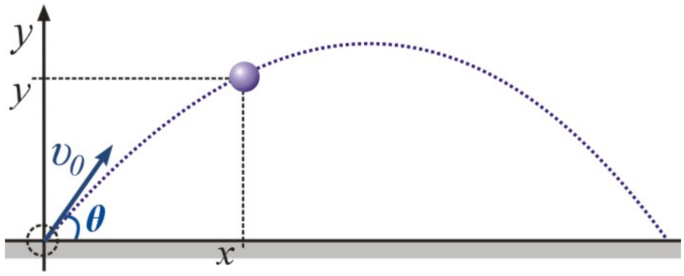
Endi esa (3.3) va (3.5) ifodalarni (III) tenglamaga qo'yamiz.

$$-mg - m\ddot{y} = 0, \ \rightarrow \ \ddot{y} = -g \quad \textbf{(3.6)}$$

Hosil bo'lgan (3.6) tenglama, yer sirtidan tik yuqoriga otilgan jism harakatining differensial tenglamasi hisoblanadi. Uni integrallash orqali umumiy holda tezlik va yo'l tenglamalarini olish mumkin.

$$\dot{y} = \dot{y}_0 - gt \quad \text{va} \quad y = y_0 + \dot{y}_0 t - \frac{gt^2}{2} \quad \textbf{(3.7)}$$

Masala 4 (gorizontga burchak ostida otilgan jism masalasi). Yer sirtidan gorizontga nisbatan biror θ burchak ostida ϑ_0 boshlang‘ich tezlik bilan otilgan jism uchun harakat tenglamasini Lagranj tenglamasi yordamida keltirib chiqaring.



Yechish: Jism otilgan nuqtani koordinata boshi deb belgilab, bu nuqtadan *Ox* va *Oy* koordinata o‘qlarini o‘tkazamiz (rasmga qarang). Ixtiyoriy vaqt onidagi jismning kinetik va potensial energiyalar

$$K=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right) \quad \text{va} \quad U=mgy \tag{4.1}$$

bo‘ladi. Bunda Lagranj funksiyasi

$$L=K-U=\frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-mgy \tag{4.2}$$

bo‘ladi. (III) Lagranj tenglamasidagi xususiy hosilalarni topamiz.

$$\begin{cases}\dfrac{\partial L}{\partial q_1}=\dfrac{\partial L}{\partial x}=\dfrac{\partial}{\partial x}\left(\dfrac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-mgy\right)=0\\[2ex] \dfrac{\partial L}{\partial q_2}=\dfrac{\partial L}{\partial y}=\dfrac{\partial}{\partial y}\left(\dfrac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-mgy\right)=-mg\end{cases} \tag{4.3}$$

$$\begin{cases}\dfrac{\partial L}{\partial \dot{q}_1}=\dfrac{\partial L}{\partial \dot{x}}=\dfrac{\partial}{\partial \dot{x}}\left(\dfrac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-mgy\right)=m\dot{x}\\[2ex] \dfrac{\partial L}{\partial \dot{q}_2}=\dfrac{\partial L}{\partial \dot{y}}=\dfrac{\partial}{\partial \dot{y}}\left(\dfrac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-mgy\right)=m\dot{y}\end{cases} \tag{4.4}$$

$$\begin{cases}\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{q}_1}\right)=\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}}\right)=\dfrac{d}{dt}(m\dot{x})=m\ddot{x}\\[2ex] \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{q}_2}\right)=\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{y}}\right)=\dfrac{d}{dt}(m\dot{y})=m\ddot{y}\end{cases} \tag{4.5}$$

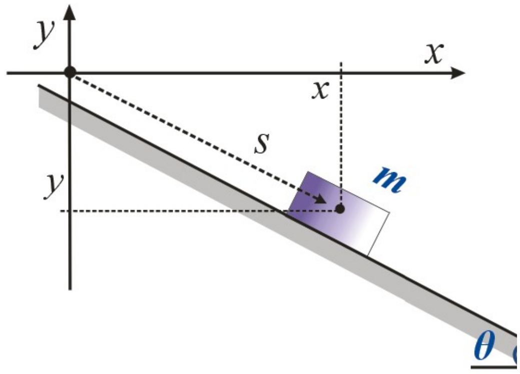
Endi esa (4.3) va (4.5) ifodalarni (III) tenglamaga qo‘yamiz.

$$\begin{cases}0-m\ddot{x}=0\\ -mg-m\ddot{y}=0\end{cases}, \quad \rightarrow \quad \begin{cases}\ddot{x}=0\\ \ddot{y}=-g\end{cases} \tag{4.6}$$

Hosil bo‘lgan (4.6) tenglama, yer sirtidan tik yuqoriga otilgan jism harakatining differensial tenglamasi hisoblanadi. Uni integrallash orqali umumiy holda tezlik va harakat tenglamalarini olish mumkin.

$$\begin{cases}\dot{x}=\dot{x}_0=\vartheta_0\cos\alpha\\ \dot{y}=\dot{y}_0-gt=\vartheta_0\sin\alpha-gt\end{cases} \quad \text{va} \quad \begin{cases}x=x_0+\vartheta_0\cos\alpha t\\ y=y_0+\vartheta_0\sin\alpha t-\dfrac{gt^2}{2}\end{cases} \tag{4.7}$$

Masala 5 (silliq qiya sirtdan tushayotgan jism masalasi). Gorizontga nisbatan qiyalik burchagi α bo‘lgan silliq qiya sirtda sirpanib tushayotgan jism uchun harakat tenglamasini Lagranj tenglamasi yordamida keltirib chiqaring.



Yechish: Dastlab, jismni qiya sirtning tepasida ushlab turilibdi deb, shu joydan koordinata boshini o‘rnatamiz hamda Ox va Oy o‘qlarini o‘tkazamiz (rasmga qarang). Qiyalik mutlaqo silliq bo‘lgani uchun qo‘yib yuborilgan jism pastga tomon qiyalik bo‘ylab sirpanib tusha boshlaydi. Uning yurgan yo‘lini s bilan belgilaymiz. Shunda kinetik va potensial energiyalar

$$K=\frac{1}{2}m\dot{s}^2 \quad \text{va} \quad U=mgy=-mgs\sin\alpha \tag{5.1}$$

bo‘ladi. Sirpanib tushayotgan jism uchun Lagranj funksiyasi

$$L=K-U=\frac{1}{2}m\dot{s}^2+mgs\sin\alpha \tag{5.2}$$

bo‘ladi. (III) Lagranj tenglamasidagi xususiy hosilalarni topamiz.

$$\frac{\partial L}{\partial q}=\frac{\partial L}{\partial s}=\frac{\partial}{\partial s}\left(\frac{1}{2}m\dot{s}^2+mgs\sin\alpha\right)=mg\sin\alpha \tag{5.3}$$

$$\frac{\partial L}{\partial \dot{q}}=\frac{\partial L}{\partial \dot{s}}=\frac{\partial}{\partial \dot{s}}\left(\frac{1}{2}m\dot{s}^2+mgs\sin\alpha\right)=m\dot{s} \tag{5.4}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)=\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{s}}\right)=\frac{d}{dt}(m\dot{s})=m\ddot{s} \tag{5.5}$$

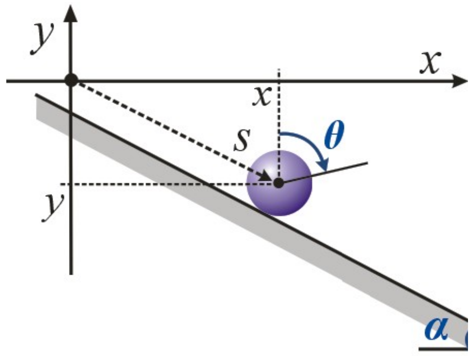
Endi esa (5.3) va (5.5) ifodalarni (III) tenglamaga qo‘yamiz.

$$mg\sin\alpha-m\ddot{s}=0, \ \rightarrow \ \ddot{s}=g\sin\alpha \tag{5.6}$$

Hosil bo‘lgan (5.6) tenglama, silliq qiyalikdan sirpanib tushayotgan jism harakatining differensial tenglamasi hisoblanadi. Uni integrallash orqali umumiy holda tezlik va yo‘l tenglamalarini olish mumkin.

$$\dot{s}=\dot{s}_0+g\sin\alpha t \quad \text{va} \quad s=s_0+\dot{s}_0t+\frac{g\sin\alpha t^2}{2} \tag{5.7}$$

Masala 6 (qiya sirtdan dumalab tushayotgan jism masalasi). Gorizontga nisbatan qiyalik burchagi α bo‘lgan qiya sirtdan dumalab tushayotgan jism uchun harakat tenglamasini Lagranj tenglamasi yordamida keltirib chiqaring.



Yechish: Dumalovchi jism qiyalik tepasidan tushayotgada uning harakatini kuzatish boshlangan nuqtani koordinata boshi sifatida olib, shu nuqtadan jism yurgan yo‘lni s bilan belgilaymiz (rasmga qarang). Bunda bosib o‘tilgan s yo‘l hamda jismning dumalashdagi θ burilish burchagi orasida

$$s = R\theta \tag{6.1}$$

ko‘rinishdagi bog‘lanish bor. Bundan hosila olish orqali

$$\dot{s} = R\dot{\theta}, \quad \ddot{s} = R\ddot{\theta} \tag{6.2}$$

bog‘lanishlarga o‘tish mumkin. Kinetik va potensial energiyalarni topamiz.

$$K = \frac{1}{2}m\dot{s}^2 + \frac{1}{2}I\dot{\theta}^2 = \frac{1}{2}m\dot{s}^2 + \frac{1}{2}I\frac{\dot{s}^2}{R^2} = \frac{1}{2}\left(m + \frac{I}{R^2}\right)\dot{s}^2 \tag{6.3}$$

$$U = mgy = -mgs\sin\alpha \tag{6.4}$$

Dumalayotgan jism uchun Lagranj funksiyasi

$$L = K - U = \frac{1}{2}\left(m + \frac{I}{R^2}\right)\dot{s}^2 + mgs\sin\alpha \tag{6.5}$$

bo‘ladi. (III) Lagranj tenglamasidagi xususiy hosilalarni topamiz.

$$\frac{\partial L}{\partial q} = \frac{\partial L}{\partial s} = \frac{\partial}{\partial s}\left(\frac{1}{2}\left(m + \frac{I}{R^2}\right)\dot{s}^2 + mgs\sin\alpha\right) = mg\sin\alpha \tag{6.6}$$

$$\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial \dot{s}} = \frac{\partial}{\partial \dot{s}}\left(\frac{1}{2}\left(m + \frac{I}{R^2}\right)\dot{s}^2 + mgs\sin\alpha\right) = \left(m + \frac{I}{R^2}\right)\dot{s} \tag{6.7}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{s}}\right) = \frac{d}{dt}\left(\left(m + \frac{I}{R^2}\right)\dot{s}\right) = \left(m + \frac{I}{R^2}\right)\ddot{s} \tag{6.8}$$

Endi esa (6.6) va (6.8) ifodalarni (III) tenglamaga qo‘yamiz.

$$mg\sin\alpha - \left(m + \frac{I}{R^2}\right)\ddot{s} = 0, \quad \to \quad \ddot{s} = \frac{mg\sin\alpha}{m + \frac{I}{R^2}} = \frac{g\sin\alpha}{1 + \frac{I}{mR^2}} = \frac{g\sin\alpha}{1+\beta} \tag{6.9}$$

Hosil bo‘lgan differensial tenglama s yo‘lga nisbatan differensial tenglama hisoblanadi. Bu yerda $\beta = \frac{I}{mR^2}$ – o‘lchamsiz va 1 dan kichik koeffitsiyent bo‘lib, uning qiymati disk yoki slindr uchun $\beta = \frac{1}{2}$ ga, xalqa yoki yupqa devorli truba uchun $\beta=1$ ga, shar uchun $\beta = \frac{3}{5}$ ga, sfera ucun $\beta = \frac{2}{3}$ ga teng. β koeffitsiyentning bu qiymatlarni e’tiborga olsak, (6.9) formula mos holda quyidagi ko‘rinishlarga o‘tadi:

$$\ddot{s} = \frac{2}{3} g \sin\alpha, \quad \ddot{s} = \frac{1}{2} g \sin\alpha, \quad \ddot{s} = \frac{5}{8} g \sin\alpha, \quad \ddot{s} = \frac{3}{5} g \sin\alpha \qquad (6.10)$$

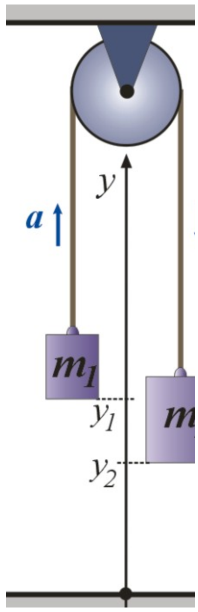
(6.2) va (6.9) formulalardan foydalanib, jismning dumalashdagi θ burilish burchagiga nisbatan differensial tenglamani hosil qilish mumkin.

$$\ddot{\theta} = \frac{g \sin\alpha}{(1+\beta) R} \qquad (6.11)$$

Bundan disk, xalqa, shar va sfera cuhun quyidagi formulani olish mumkin:

$$\ddot{\theta} = \frac{2g}{3R} \sin\alpha, \quad \ddot{\theta} = \frac{g}{2R} \sin\alpha, \quad \ddot{\theta} = \frac{5g}{8R} \sin\alpha, \quad \ddot{\theta} = \frac{3g}{5R} \sin\alpha \qquad (6.12)$$

(6.9) yoki (6.10) tenglamalarni integrallash natijasida dumlayotgan jismning tezlik va yo‘l tenglamalari hosil qilinadi. (6.11) yoki (6.12) tenglamalarni integrallash natijasida esa dumlayotgan jismning burchak tezlik va burilish burchagi tenglamalari hosil qilinadi.



Masala 7 (atvud mashinasi masalasi). Qo‘zg‘almas yengil blokka massalari m_1 va m_2 ($m_2 > m_1$) massali jismlar osilgan. Jismlar uchun harakat tenglamasini Lagranj tenglamasidan foydalanib yeching.

Yechish: Yerdan biror balandlikka osilgan yengil blokka uzunligi ℓ ga teng bo‘lgan yengil va cho‘zilmas ip o‘tkazilgan va uning uchlariga massalari m_1 va m_2 ga teng bo‘lgan ikkita jism osilgan deb olaylik. Yer sirtida koordinata boshini olib Oy o‘qini tik holda tepaga yo‘naltiramiz. Jismlarning koordinatalarini mos holda y_1 va y_2 bilan belgilaymiz. Blok aylanganda jismlarning biri ko‘tarilganda ikkinchisi tushgani uchun

$$y_1 + y_2 + \ell = const, \quad \rightarrow \quad y_2 = const - \ell - y_1 = C_1 - y_1 \qquad (7.1)$$

munosabat o‘rinlidir (rasmga qarang). Bundan esa

$$\dot{y}_2 = -\dot{y}_1, \quad \ddot{y}_2 = -\ddot{y}_1 \tag{7.2}$$

kelib chiqadi. Kinetik va potensial energiyalarni topamiz.

$$K = \frac{1}{2}m_1\dot{y}_1^2 + \frac{1}{2}m_2\dot{y}_2^2 = \frac{1}{2}(m_1 + m_2)\dot{y}_1^2 \tag{7.3}$$

$$U = m_1gy_1 + m_2gy_2 = m_1gy_1 - m_2gy_1 + m_2gC_1 \tag{7.4}$$

Jismlar sistemasi uchun Lagranj funksiyasi

$$L = K - U = \frac{1}{2}(m_1 + m_2)\dot{y}_1^2 + (m_2 - m_1)gy_1 - m_2gC_1 \tag{7.5}$$

bo‘ladi. (III) Lagranj tenglamasidagi xususiy hosilalarni topamiz.

$$\frac{\partial L}{\partial q} = \frac{\partial L}{\partial y_1} = \frac{\partial}{\partial y_1}\left(\frac{1}{2}(m_1 + m_2)\dot{y}_1^2 + (m_2 - m_1)gy_1 - m_2gC_1\right) = (m_2 - m_1)g \tag{7.6}$$

$$\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial \dot{y}_1} = \frac{\partial}{\partial \dot{y}_1}\left(\frac{1}{2}(m_1 + m_2)\dot{y}_1^2 + (m_2 - m_1)gy_1 - m_2gC_1\right) = (m_1 + m_2)\dot{y}_1 \tag{7.7}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_1}\right) = \frac{d}{dt}((m_1 + m_2)\dot{y}_1) = (m_1 + m_2)\ddot{y}_1 \tag{7.8}$$

Endi esa (7.6) va (7.8) ifodalarni (III) tenglamaga qo‘yamiz.

$$(m_2 - m_1)g - (m_1 + m_2)\ddot{y}_1 = 0, \rightarrow \ddot{y}_1 = \frac{m_2 - m_1}{m_1 + m_2}g \tag{7.9}$$

Hosil bo‘lgan (7.9) tenglama, birinchi jism harakatining differensial tenglamasi hisoblanadi. Ikkinchi jismning differensial tenglamasi quyidagicha:

$$\ddot{y}_2 = -\ddot{y}_1 = -\frac{m_2 - m_1}{m_1 + m_2}g \tag{7.10}$$

(7.9) hamda (7.10) tenglamalarni integrallash orqalihar bir jism uchun umumiy holda tezlik va yo‘l tenglamalarini olish mumkin.

$$\begin{cases} \dot{y}_1 = \dot{y}_{1.0} + \dfrac{m_2 - m_1}{m_1 + m_2}gt \\ \dot{y}_2 = \dot{y}_{2.0} - \dfrac{m_2 - m_1}{m_1 + m_2}gt \end{cases} \quad \text{va} \quad \begin{cases} y_1 = y_{1.0} + \dot{y}_{1.0}t + \dfrac{m_2 - m_1}{m_1 + m_2}\cdot\dfrac{gt^2}{2} \\ \dot{y}_2 = y_{2.0} + \dot{y}_{2.0}t - \dfrac{m_2 - m_1}{m_1 + m_2}\cdot\dfrac{gt^2}{2} \end{cases} \tag{7.11}$$

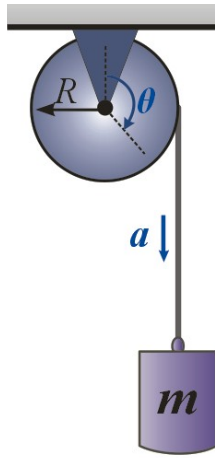
Bunda albatta boshlang‘ich tezlik va boshlang‘ich koordinatalar uchun

$$\dot{y}_{2.0} = -\dot{y}_{1.0}, \quad y_{2.0} = -y_{1.0} \tag{7.12}$$

munosobatni e’tiborga olish kerak bo‘ladi.

Masala 8 (qo‘zg‘almas blokka osilgan jism masalasi). Qo‘zg‘almas slindrik blokka o‘ralgan ipning bir uchiga *m* massali yuk osilgan. Blokning massasi *M*, inersiya momenti *I*, radiusi *R* ga teng. Yuk qo‘yib yuborilgandan keyin uning harakat tenglamasi qanday ko‘rinishda bo‘ladi. Masalani Lagranj tenglamasidan foydalanib yeching.

Yechish: Qo‘zg‘almas blok markazini koordinata boshi sifatida olib, shu nuqtadan boshlab jism pastga tushgan yo‘lni *s* bilan belgilaymiz (rasmga qarang).



Bunda bosib o‘tilgan *s* yo‘l hamda blokning aylanishidagi θ burilish burchagi orasida

$$s = R\theta \tag{8.1}$$

ko‘rinishdagi bog‘lanish bor. Bundan hosila olish orqali

$$\dot{s} = R\dot{\theta}, \quad \ddot{s} = R\ddot{\theta} \tag{8.2}$$

bog‘lanishlarga o‘tish mumkin. Qo‘zg‘almas blok va unga ip orqali osilgan jismdan iborat sistemaning kinetik va potensial energiyalarni topamiz.

$$K = \frac{1}{2}m\dot{s}^2 + \frac{1}{2}I\dot{\theta}^2 = \frac{1}{2}m\dot{s}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{\dot{s}}{R}\right)^2 = \frac{1}{2}\left(m + \frac{M}{2}\right)\dot{s}^2 \tag{8.3}$$

$$U = mgy = -mgs \tag{8.4}$$

Dumalayotgan jism uchun Lagranj funksiyasi

$$L = K - U = \frac{1}{2}\left(m + \frac{M}{2}\right)\dot{s}^2 + mgs \tag{8.5}$$

bo‘ladi. (III) Lagranj tenglamasidagi xususiy hosilalarni topamiz.

$$\frac{\partial L}{\partial q} = \frac{\partial L}{\partial s} = \frac{\partial}{\partial s}\left(\frac{1}{2}\left(m + \frac{M}{2}\right)\dot{s}^2 + mgs\right) = mg \tag{8.6}$$

$$\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial \dot{s}} = \frac{\partial}{\partial \dot{s}}\left(\frac{1}{2}\left(m + \frac{M}{2}\right)\dot{s}^2 + mgs\right) = \frac{1}{2}\left(m + \frac{M}{2}\right)\dot{s} \tag{8.7}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{s}}\right) = \frac{d}{dt}\left(\frac{1}{2}\left(m + \frac{M}{2}\right)\dot{s}\right) = \frac{1}{2}\left(m + \frac{M}{2}\right)\ddot{s} \tag{8.8}$$

Endi esa (6.6) va (6.8) ifodalarni (III) tenglamaga qo‘yamiz.

$$mg - \frac{1}{2}\left(m + \frac{M}{2}\right)\ddot{s} = 0, \ \rightarrow \ \ddot{s} = \frac{4m}{2m + M}g \tag{8.9}$$

Hosil bo‘lgan differensial tenglama s yo‘lga nisbatan differensial tenglama hisoblanadi. (8.2) va (8.9) formulalardan foydalanib, qo‘zg‘almas blokning aylanishdagi θ burilish burchagiga nisbatan differensial tenglamani hosil qilish mumkin.

$$\ddot{\theta} = \frac{4m}{2m+M}\frac{g}{R} \tag{8.10}$$

(8.9) tenglamani integrallash natijasida tushayotgan jismning tezlik va yo‘l tenglamalari hosil qilinadi. (8.10) tenglamani integrallash natijasida esa aylanayotgan qo‘zg‘almas blokning burchak tezlik va burilish burchagi tenglamalari hosil qilinadi.

Shunday qilib, mazkur maqolada mexanika masalalarini talabalarga ko‘nikma bo‘lib qolgan an’anaviy usul – Nyuton qonunlari yordamida emas, balki Lagranj tenglamasi hamda Lagranj funksiyalarini qo‘llagan holda yechishga oid namuna masalalar ishlab ko‘rsatib berdik. Bunday noan’anaviy usul yordamida masalalar yechish talabalar uchun juda ham foydali bo‘lib, ularning fikrlash doirasini kengaytirishga, hodisa-jarayonlar haqida to‘laqonli bilimga ega bo‘lishga, bilimini oshirishga hamda turli yo‘llar bilan masalalarni ishlash ko‘nikmasini shakllanishiga olib keladi.

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